A Game-Theoretic Perspective on Risk-Sensitive Reinforcement Learning

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Reinforcement Learning (RL)

\[ J(\pi) := \sum_{t=0}^{\infty} \gamma^t R_t \]
Classical RL objective: expectation maximization

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Good objective for the Arcade Learning Environment (Bellemare et al., 2013)
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Unfit for a risky task like clinical treatment suggestion!
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Conditional-value-at-risk RL: a risk-sensitive objective

$$CVaR_\alpha(Z) = \mathbb{E}[z \mid z \leq \text{VaR}_\alpha(Z)]$$

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All existing approaches require distributional RL methods.
Game Structure

Antagonist produces perturbed next state transitions $\hat{P}_t$ to minimize the protagonist's rewards
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Antagonist has limited budget

$$\delta_0(s_1)\delta_1(s_2) \cdots \delta_{T-1}(s_T) \leq \eta$$
Retrieving CVaR RL optimal policies

Max-min objective: $\max_{\pi} \min_{\Lambda} \mathbb{E}[J^\eta(\pi, \Lambda)]$
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\max_\pi \min_\Lambda \mathbb{E}[J^\eta(\pi, \Lambda)]
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The solution is the equilibrium point \((\pi^*, \Lambda^*)\), for which we have (Chow et al., 2015):
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\pi^* = \arg \max_\pi \text{CVaR}_{\frac{1}{\eta}} \left[ J(\pi) \right]
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CARL: Game properties

The objective for both the agent and the adversary is to maximize their expected rewards.
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Risk tolerance is based on a single hyperparameter and is easy to interpret.
Stackelberg games for gradient updates

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Stackelberg game: a leader ($\pi$) takes for granted that its follower ($\Lambda$) is optimal with respect to itself.

$$\pi^* = \arg \max_{\pi} \left\{ \mathbb{E} [J^n(\pi, \Lambda')] \text{ s.t. } \Lambda' = \arg \max_{\Lambda} \mathbb{E} [J^n(\pi, \Lambda)] \right\}$$

$$\Lambda^* = \arg \max_{\Lambda} \mathbb{E} [J^n(\pi, \Lambda)]$$
Practical Stackelberg-based algorithm

Algorithm 1: CVaR Adversarial Stackelberg Algorithm

Require: $\pi_\theta$ (protagonist), $\Lambda_\omega$ (antagonist), $\eta$ (perturbation budget), $K_{\text{ant}}$ (number of intermediate antagonist steps)

1: $N_{\text{updates}} = 0$
2: while training not done do
3:   Get initial state $s_t$
4:   $\eta_\tau = \eta$ \hfill $\triangleright$ Remaining antagonist budget
5:   while $s_t$ not terminal do
6:     $a_t \sim \pi_\theta(s_t), \mathcal{P}_t = \mathcal{P}(s_t, a_t)$
7:     $\delta_t = \Lambda_\omega(\mathcal{P}_t, \eta_\tau)$
8:     $\hat{\mathcal{P}}_t = \mathcal{P}_t \circ \delta$
9:     $s_{t+1} \sim \hat{\mathcal{P}}_t, r_{t+1} \sim \mathcal{R}(s_{t+1})$
10:    $\eta_\tau = \frac{\eta_\tau}{\delta_t(s_{t+1})}$ \hfill $\triangleright$ Update remaining budget
11:   end while
12:   Update $\theta$ or $\omega$ according to $N_{\text{updates}}$ and $K_{\text{ant}}$.
13:   $N_{\text{updates}} = N_{\text{updates}} + 1$
14: end while
Risky Gridworld: experimental setting

5% chance that the environment executes a random action.

The agent’s degree of caution is represented by its willingness to move lower on the grid to distance itself from the lava tiles.
Empirical results

$\eta = 1$  \hspace{1cm} $\eta = 25$  \hspace{1cm} $\eta = 100$

Increasing the adversary's budget leads to an increasingly cautious agent.
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There appears to be instability issues in the training procedure.
Conclusion

We proposed a new risk-sensitive RL method for the CVaR risk measure which does not require distributional RL algorithms.
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We estimate that our proposal can serve as a building block because it paves the way to incorporate results from the Game Theory literature to risk-sensitivity in RL.
References

