A Game-Theoretic Perspective on Risk-Sensitive Reinforcement Learning

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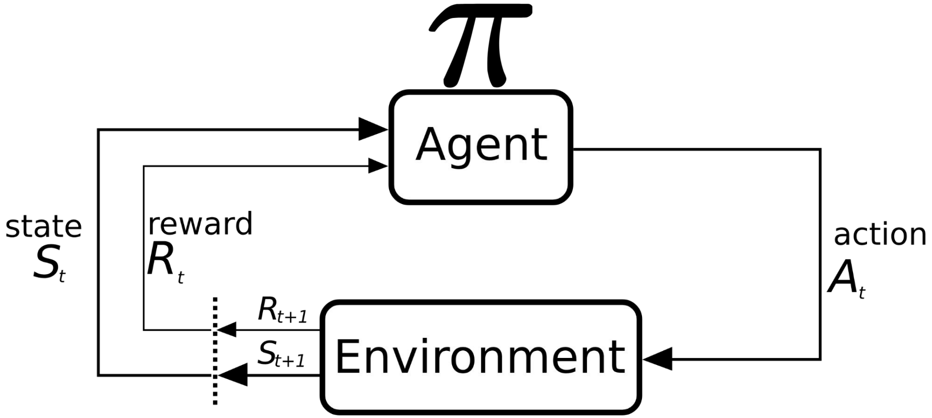
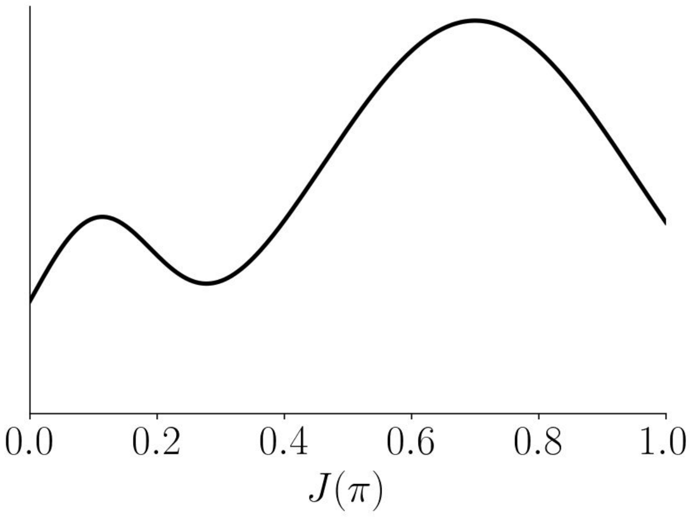
Reinforcement Learning (RL)

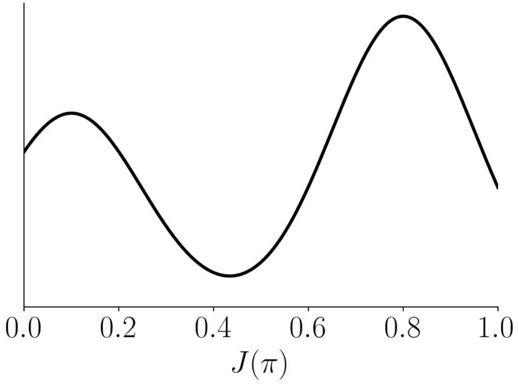


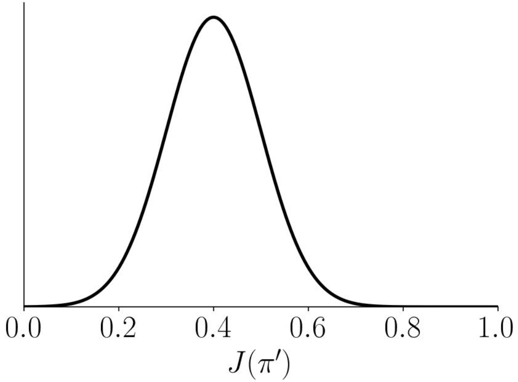
$$J(\pi) := \sum_{t=0}^{\infty} \gamma^t R_t$$



Classical RL objective: expectation maximization

$$\pi^\star = \arg\max_\pi \mathbb{E}[J(\pi)]$$





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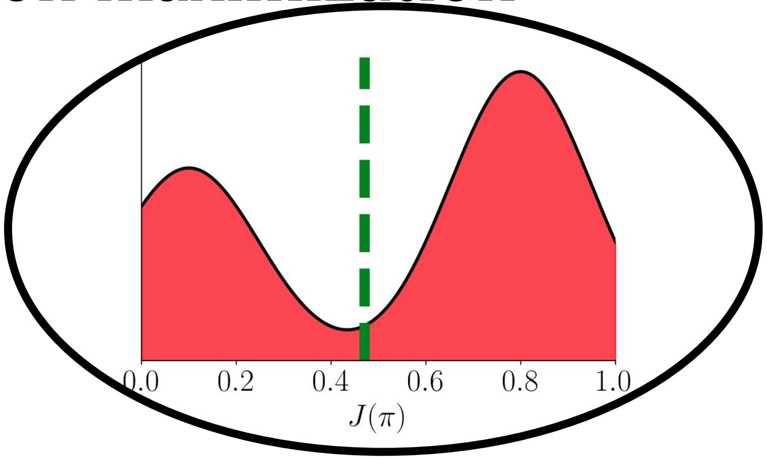
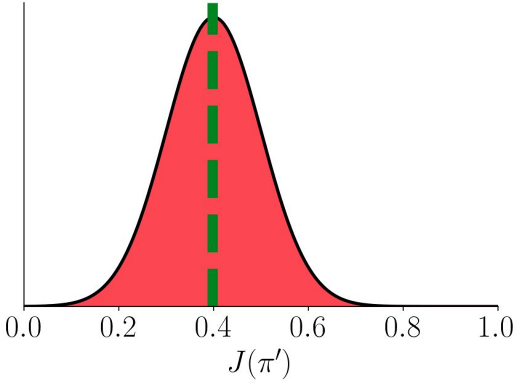
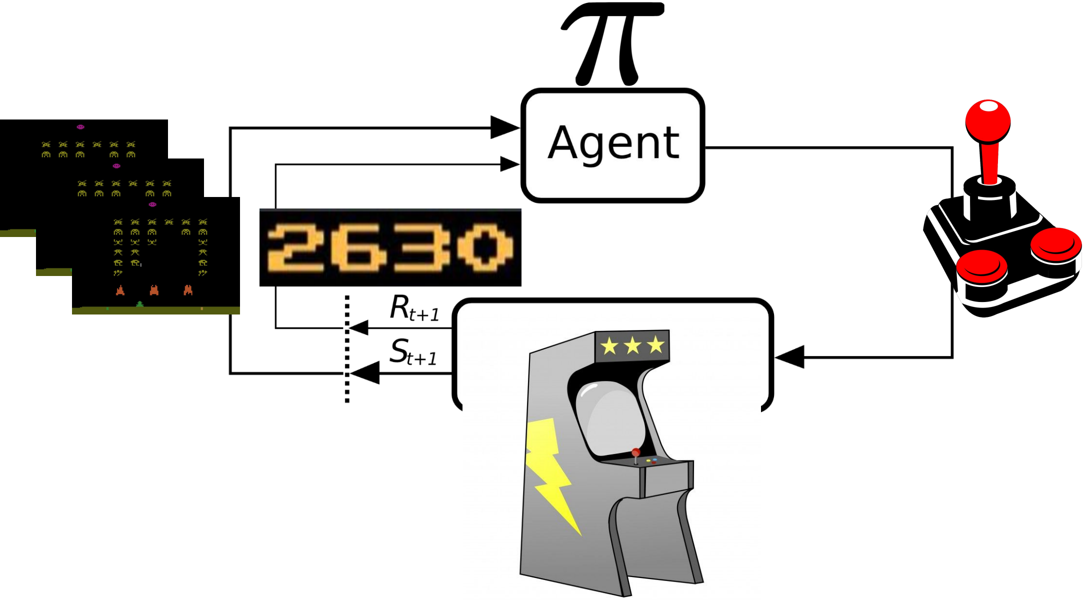
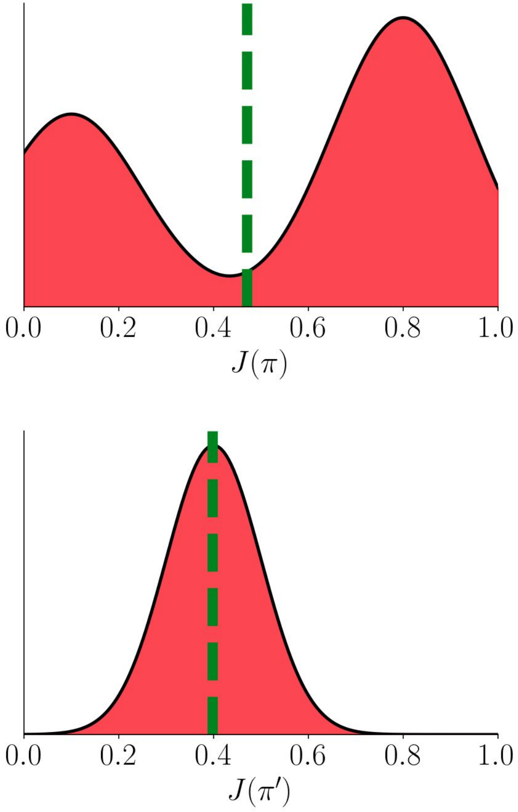




Good objective for the Arcade Learning Environment (Bellemare et al., 2013)





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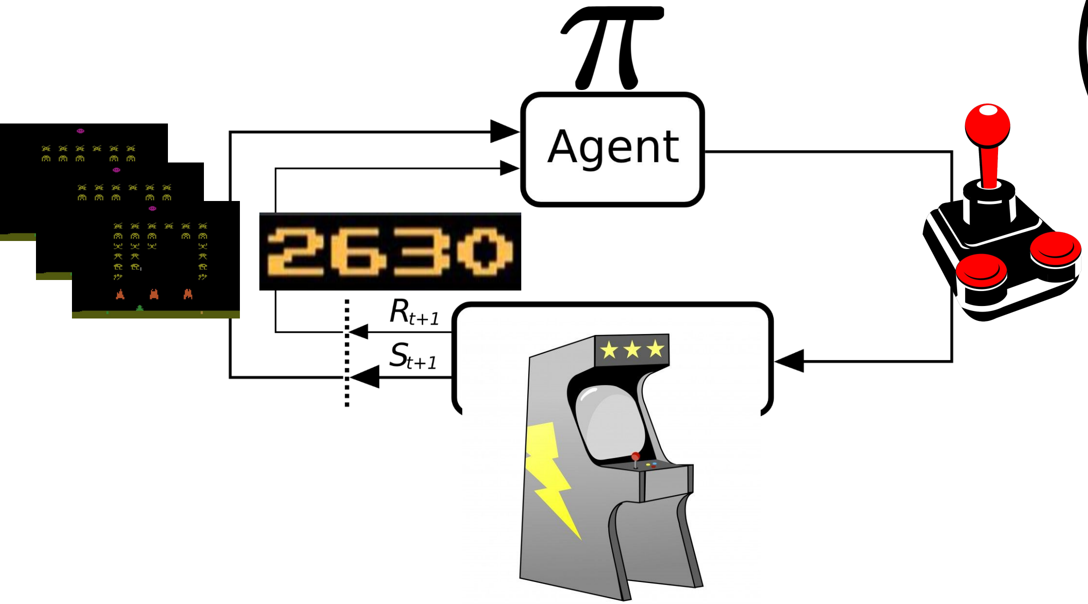

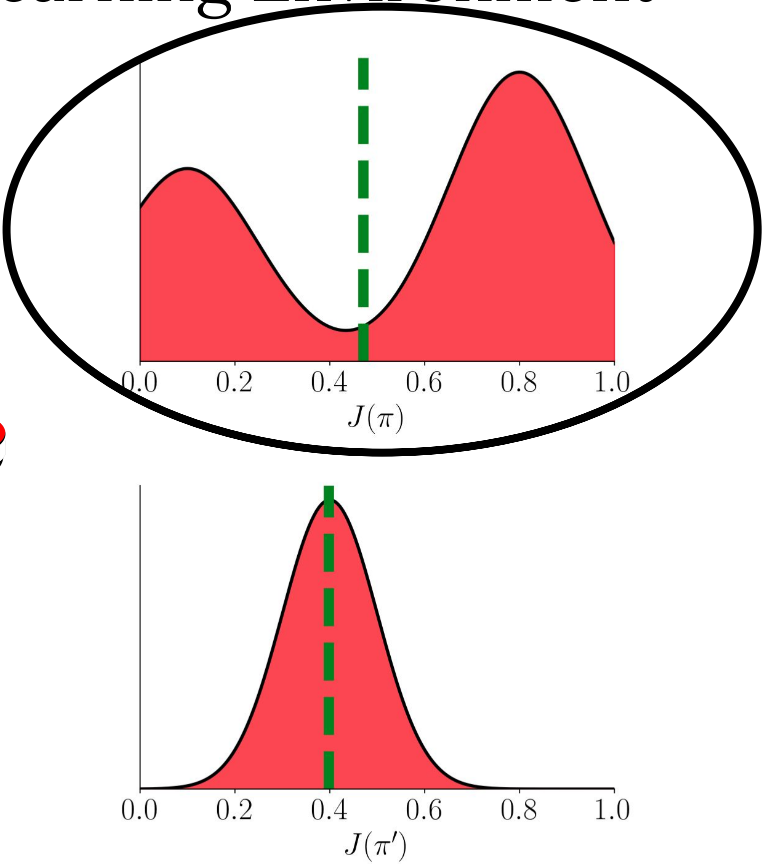

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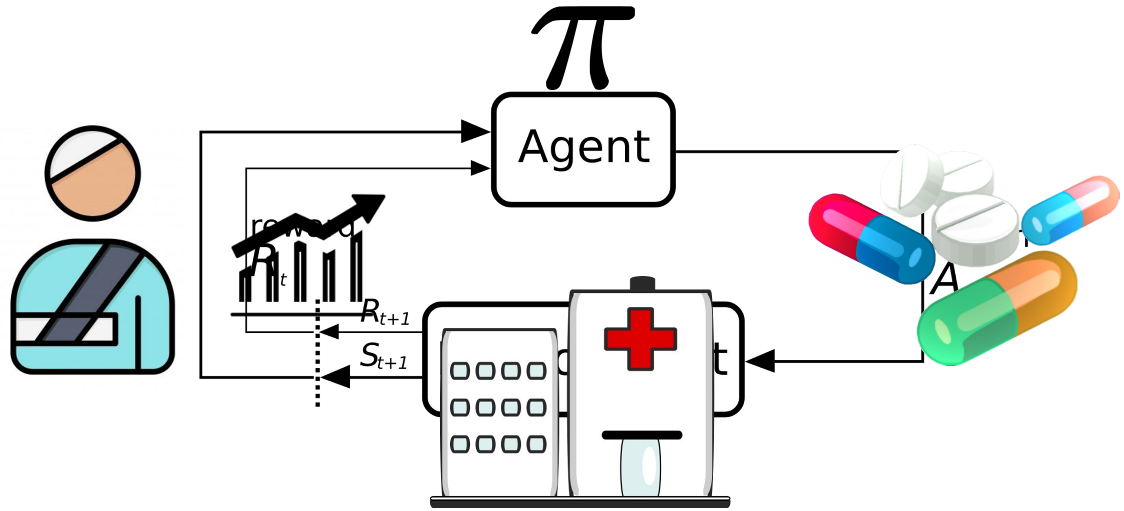

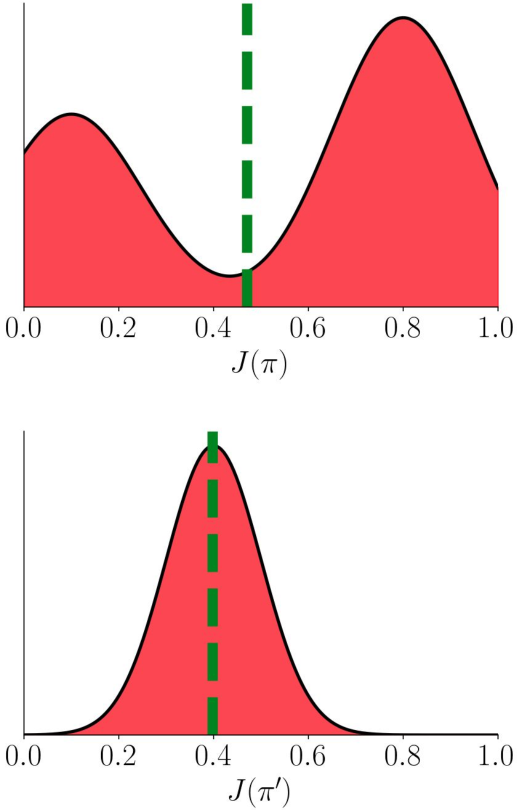

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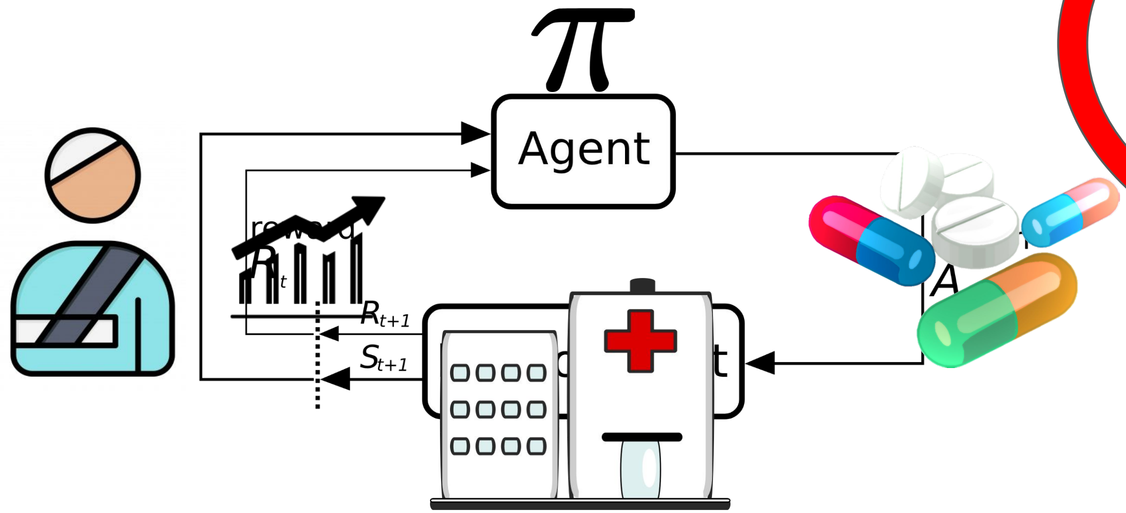
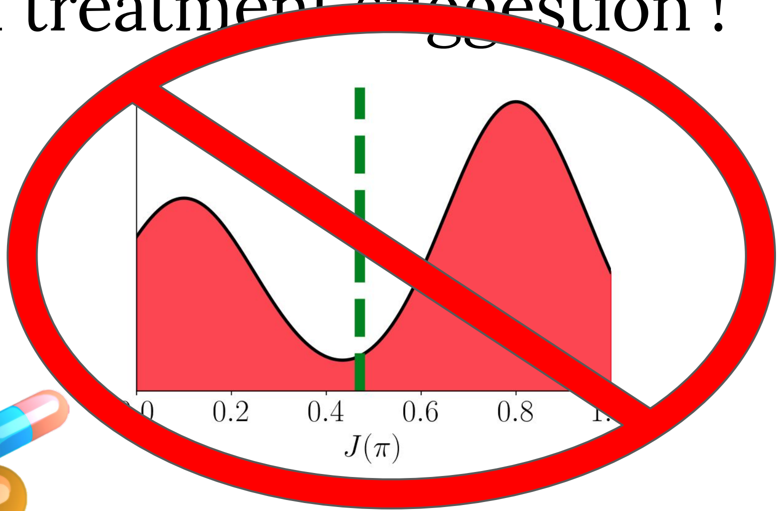
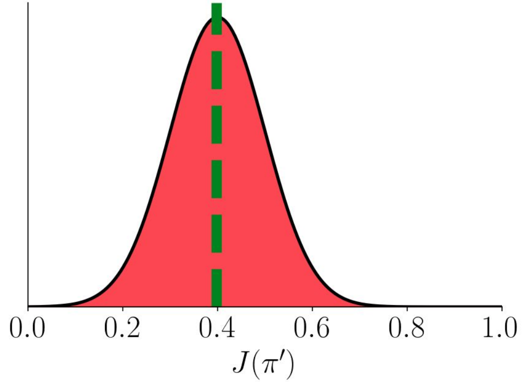


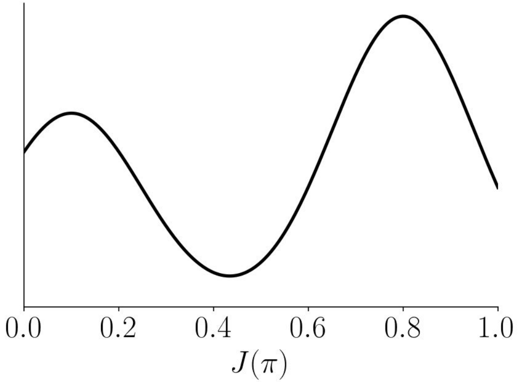


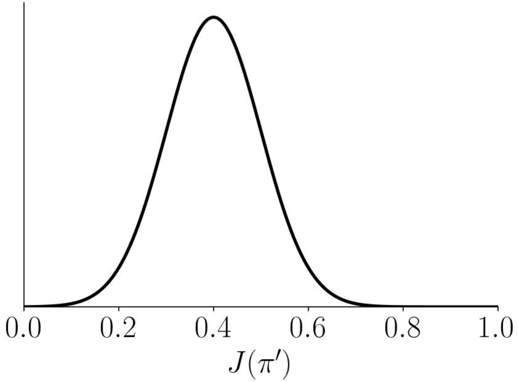


Conditional-value-at-risk RL: a risk-sensitive objective

$$\mathrm{CVaR}_\alpha(Z) = \mathbb{E}[z \mid z \leq \mathrm{VaR}_\alpha(Z)]$$

$$\pi^\star = \arg\max_\pi \mathrm{CVaR}_\alpha[J(\pi)]$$





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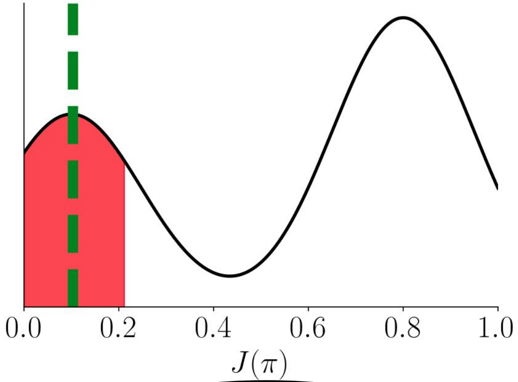

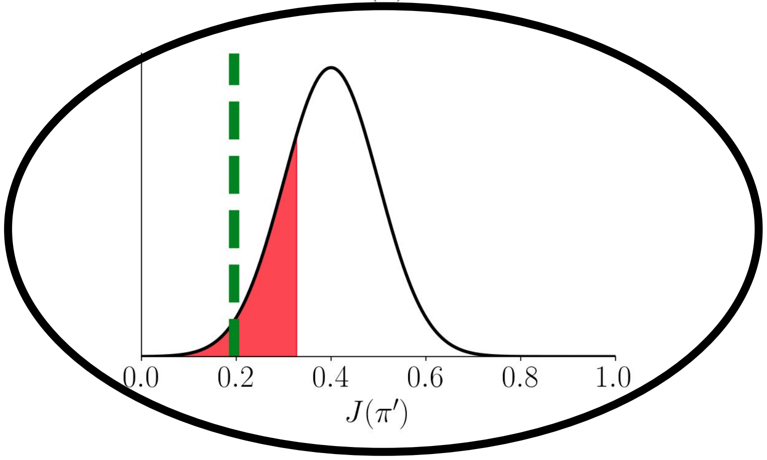

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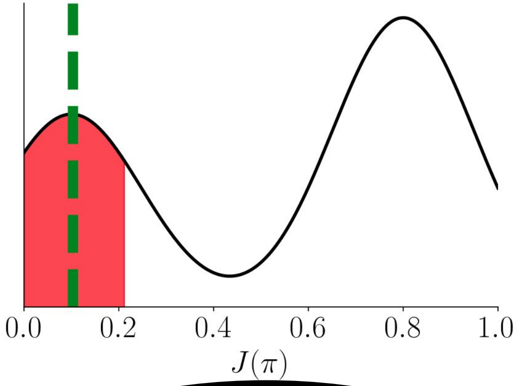
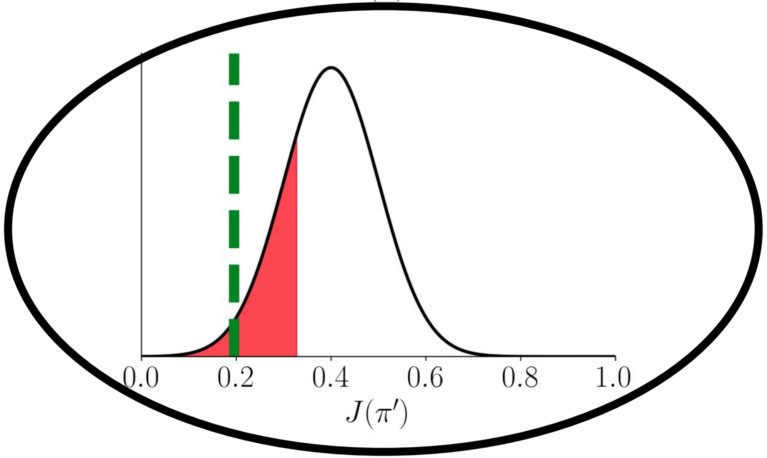
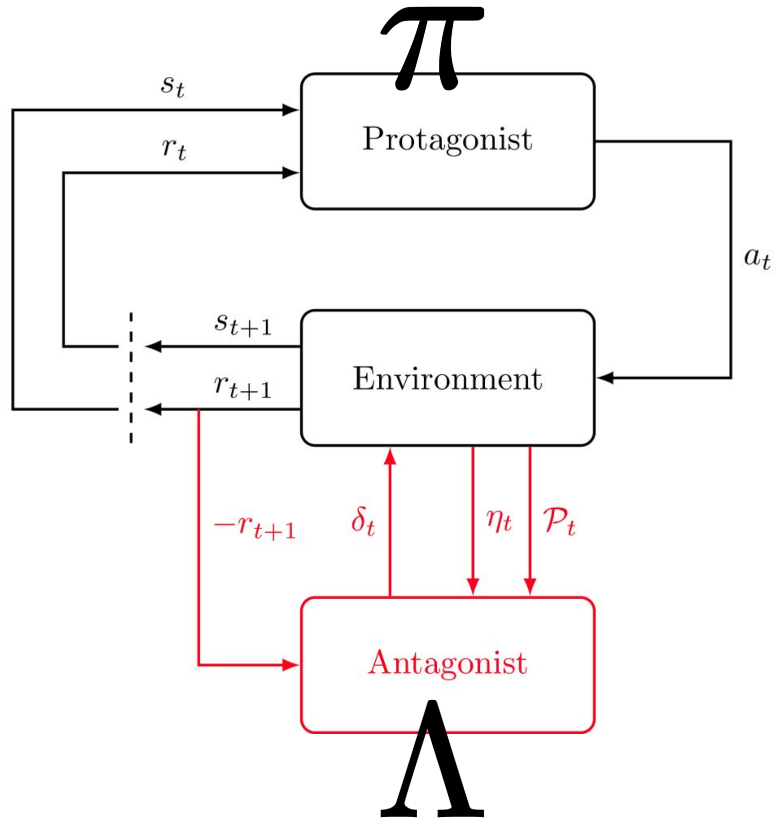




All existing approaches require distributional RL methods.

Game Structure



Antagonist produces perturbed next state transitions $\hat{P}_t$ to minimize the protagonist's rewards

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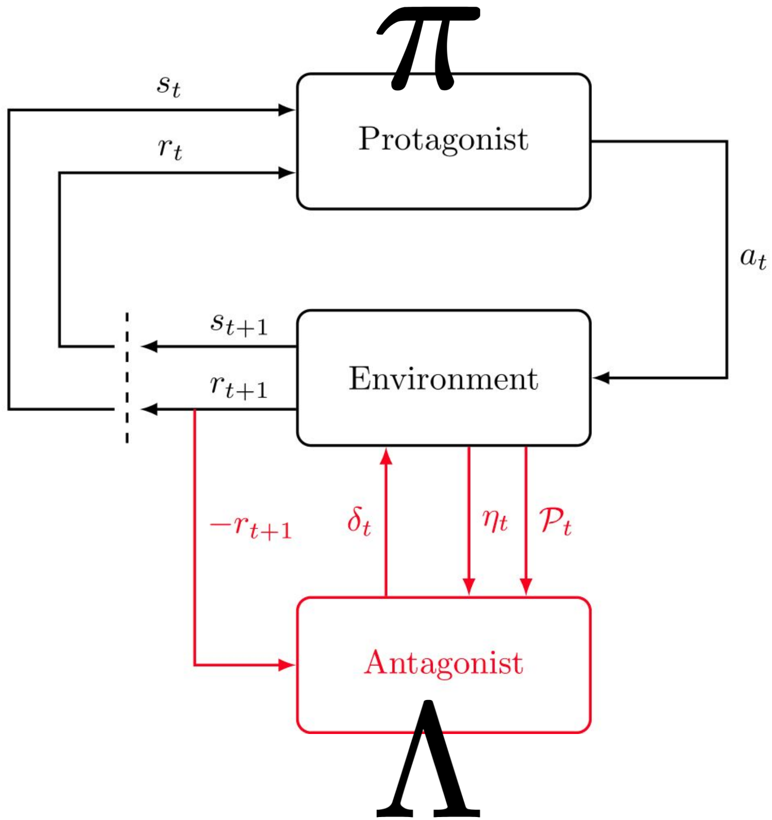

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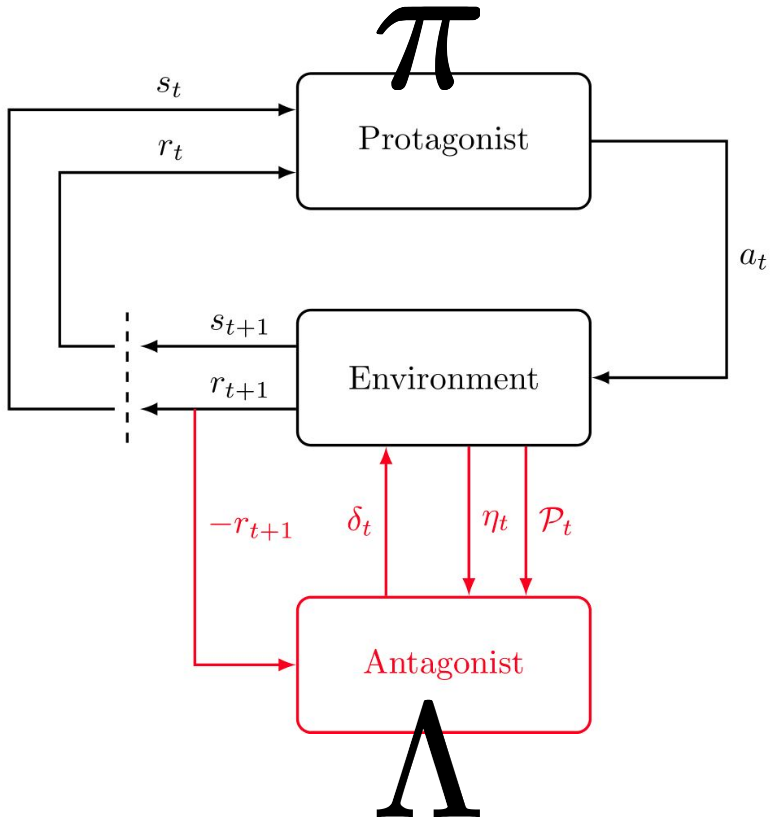

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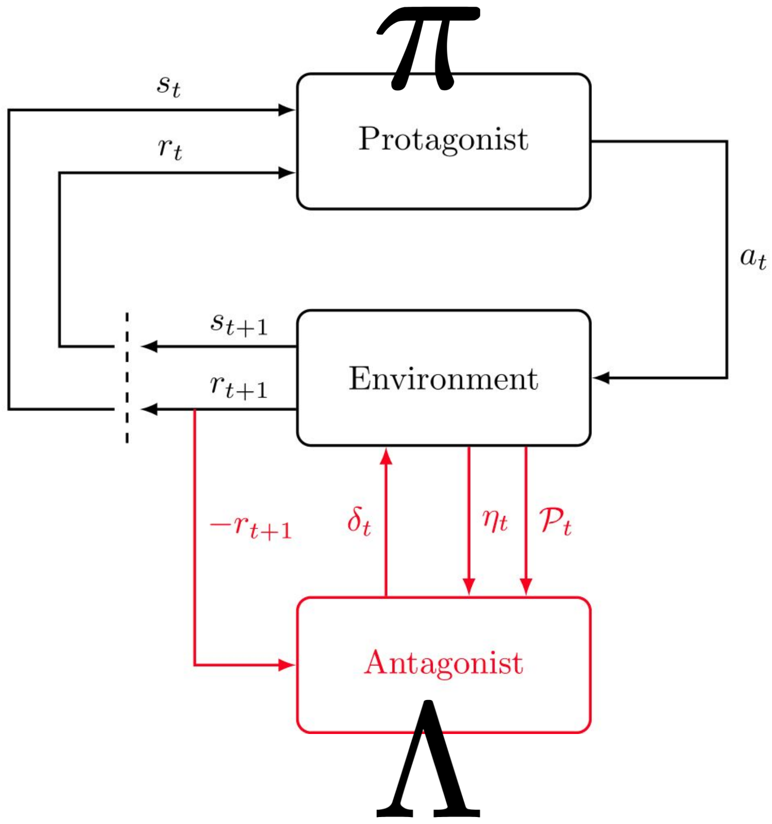

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Antagonist has limited budget

$$\delta_0(s_1)\delta_1(s_2)\cdots\delta_{T-1}(s_T) \leq \eta$$

Retrieving CVaR RL optimal policies

Max-min objective: $$\max_{\pi} \min_{\Lambda} \; \mathbb{E}[J^{\eta}(\pi, \Lambda)]$$

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The solution is the equilibrium point $(\pi^{\star}, \Lambda^{\star})$, for which we have (Chow et al., 2015):

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CARL: Game properties

The objective for both the agent and the adversary is to maximize their **expected** rewards.

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Risk tolerance is based on a single hyperparameter and is easy to interpret.

Stackelberg games for gradient updates

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Stackelberg game: a leader (π) takes for granted that its follower (Λ) is optimal with respect to itself.

$$\pi^\star = \arg\max_\pi \left\{ \mathbb{E}\left[J^\eta\left(\pi,\, \Lambda'\right)\right] \text{ s.t. } \Lambda' = \arg\max_\Lambda \mathbb{E}[J^\eta(\pi,\, \Lambda)] \right\}$$

$$\Lambda^\star = \arg\max_\Lambda \mathbb{E}[J^\eta(\pi,\, \Lambda)]$$

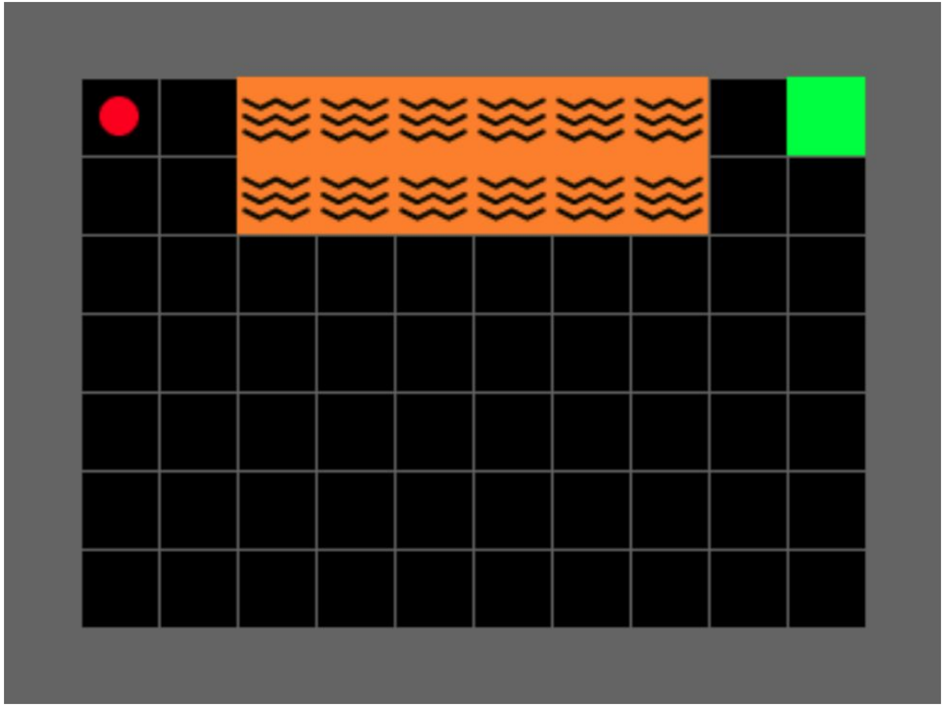
Practical Stackelberg-based algorithm

Algorithm 1: CVaR Adversarial Stackelberg Algorithm

Require: π_θ (protagonist), Λ_ω (antagonist), η (perturbation budget), K_{ant} (number of intermediate antagonist steps)

$N_{\text{updates}} = 0$
while training not done **do**
  Get initial state s_t
  $\eta_\tau = \eta$ ▷ Remaining antagonist budget
  while s_t not terminal **do**
    $a_t \sim \pi_\theta(s_t), \mathcal{P}_t = \mathcal{P}(s_t, a_t)$
    $\delta_t = \Lambda_\omega(\mathcal{P}_t, \eta_\tau)$
    $\hat{\mathcal{P}}_t = \mathcal{P}_t \circ \delta$
    $s_{t+1} \sim \hat{\mathcal{P}}_t, r_{t+1} \sim \mathcal{R}(s_{t+1})$
    $\eta_\tau = \frac{\eta_\tau}{\delta_t(s_{t+1})}$ ▷ Update remaining budget
  end while
  Update θ or ω according to N_{updates} and K_{ant}.
  $N_{\text{updates}} = N_{\text{updates}} + 1$
end while

Risky Gridworld: experimental setting

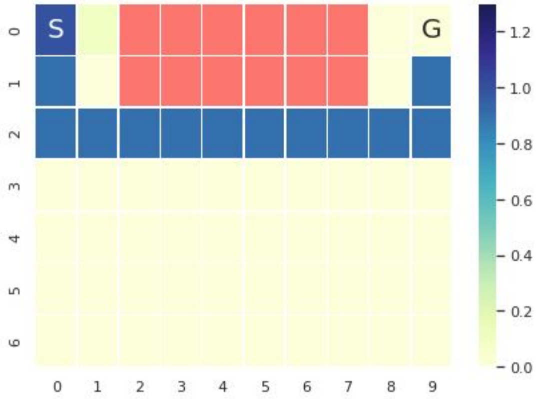


5 % chance that the environment executes a random action.

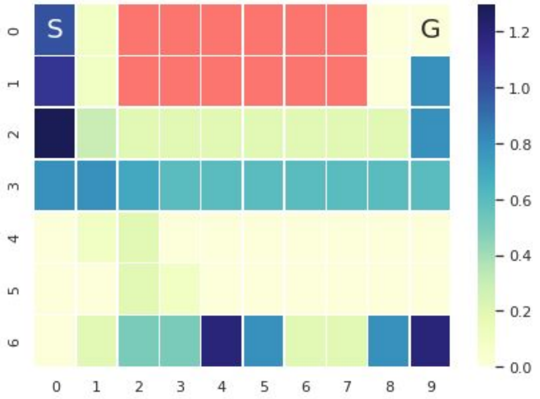
The agent's degree of caution is represented by its willingness to move lower on the grid to distance itself from the lava tiles.

Empirical results

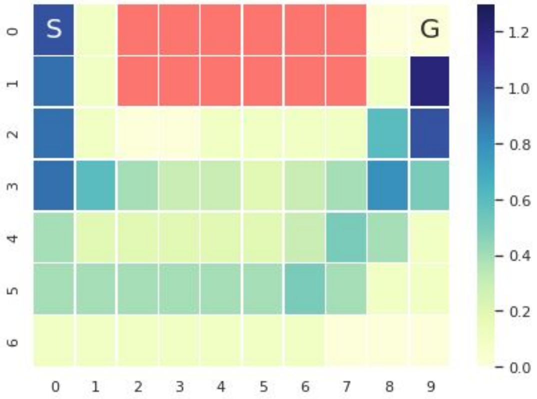
$\eta = 1$



$\eta = 25$



$\eta = 100$



Increasing the adversary’s budget leads to an increasingly cautious agent.

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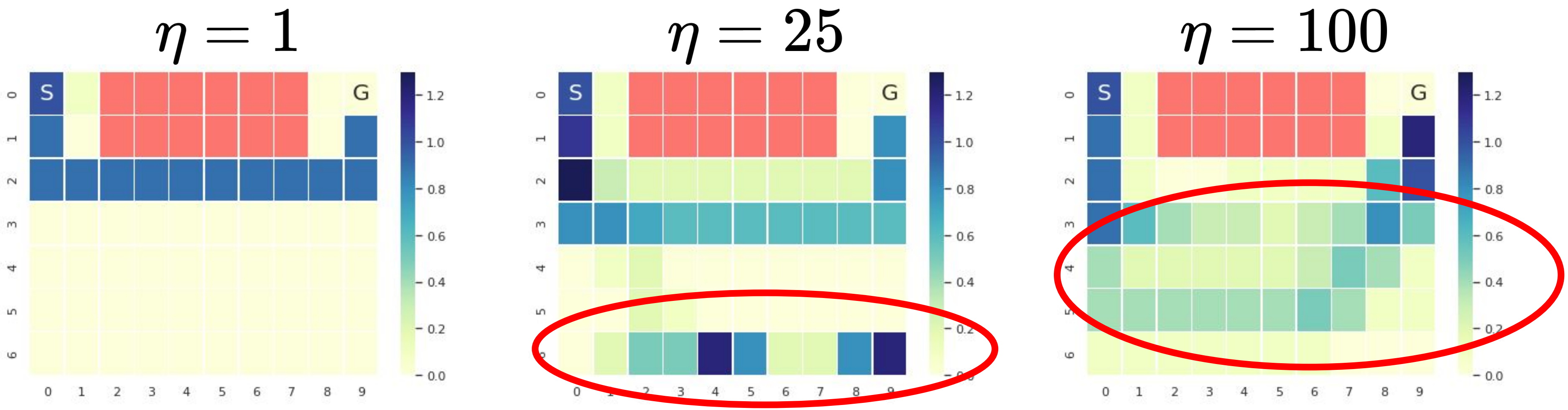

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There appears to be instability issues in the training procedure.

Conclusion

We proposed a new risk-sensitive RL method for the CVaR risk measure which does not require distributional RL algorithms.

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We proposed a new risk-sensitive RL method for the CVaR risk measure which does not require distributional RL algorithms.

We estimate that our proposal can serve as a building block because it paves the way to incorporate results from the Game Theory litterature to risk-sensitivity in RL.

References

Bellemare, M. G., Naddaf, Y., Veness, J., & Bowling, M. (2013). The arcade learning environment: An evaluation platform for general agents. Journal of Artificial Intelligence Research, 47, 253-279.

Chow, Y., Tamar, A., Mannor, S., & Pavone, M. (2015). Risk-sensitive and robust decision-making: a cvar optimization approach. arXiv preprint arXiv:1506.02188.

Fiez, T., Chasnov, B., & Ratliff, L. J. (2019). Convergence of learning dynamics in stackelberg games. arXiv preprint arXiv:1906.01217.